# Motion Coordination for Redundant Robots by Tracking Position-Level Equality Constraints 

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#### Abstract

This paper describes a novel motion coordination method for redundant robots. The method combines closed-form reverse position analysis and multi-criteria optimization to form a powerful and efficient algorithm. This method of redundancy resolution has been tested (either in simulation or experimentation) on robots with $7,8,9,10,17$, and 21 DOF. This paper presents results for a dual-arm robot with 17 DOF designed and implemented at Oak Ridge National Laboratory.


## Introduction

Motion coordination for redundant robots enjoys a rich history as a part of the inverse kinematics problem. Dimentberg in the 1950's and Freudenstein in the 1960's and 1970's were seminal authors. With the realization in the late 1960's that a serial robot could be modelled as a spatial mechanism, the disciplined and analytical theory of mechanisms was applied to the exciting new field of robotics. This work dominated inverse kinematics research during the 1970's as the search for a general closed-form solution for robots with six Degrees Of Freedom (DOF) became the "Mount Everest" of kinematics problems (Freudenstein, 1972). Duffy, Pieper, and Roth were at the forefront of inverse kinematics research during this time.

Within the context of redundant robots, the focus shifted towards optimization and linear algebra during the 1980's. Much of this work derives from Whitney's (1969) resolved motion rate control that suggests the use of the pseudoinverse to resolve redundancy. Liegeois (1977) showed the extension of this method to include self-motions via the null-space. Since then, a large number of researchers have implemented pseudo-inverse based methods. Notable approaches include: Seraji's (1992) configuration control, Baillieul's (1986) extended Jacobian, and the Jacobian transpose (Das, Slotine, and Sheridan, 1988). Dubey and Luh (1988) include task-based performance measures in
the redundancy resolution. Maciejewski (1989) discusses the kinetic limitations of redundant robots.

This paper discusses a motion coordination method that has shown great promise in both simulation and application. Essentially, the method uses closed-form reverse position analysis to satisfy the placement constraints on the robot's hand and numerical optimization to resolve the redundancy. The numerical optimization generates configuration options and, based on a six DOF substructure of the robot's geometry, closed-form reverse position analysis ensures the options satisfy the placement constraints. This process explicitly identifies configuration options within the robot's null space. A decision making process based on multiple performance criteria chooses one option as the next setpoint for the robot's servo controllers. Crane, Duffy, and Carnahan (1991) have also shown the use of closed-form reverse position analysis to solve 6 DOF substructures within a redundant robot, though they leave the decision making to a human operator.

## Constraint Tracking

Constraint tracking acts as a filter to eliminate options not satisfying the positional $\left(P_{x}, P_{y}, P_{z}\right)$ and orientational ( $\alpha, \beta, \gamma$ ) equality constraints, on the placement of the robot's EEF. Concatenation of the geometric transformations associated with each of these constraints generates the transformation, ${ }_{n}^{0} T$, the placement of the robot's hand must satisfy. The formulation of the transformation for the closed-form position analysis proceeds as follows:

$$
{ }_{n}^{0} T={ }_{n-5}^{0} T{ }_{n}^{n-5} T,
$$

and

$$
{ }_{n}^{n-5} T={ }_{n-5}^{0} T^{-1}{ }_{n}^{0} T .
$$

Given the general transformation, ${ }_{n}^{n-5} T$, the fullyconstrained reverse position solution, $\theta_{n-5}$ to $\theta_{n}$, also
satisfies ${ }_{n}^{0} T$. Figure 1. depicts the geometry of these transformations.


Figure 1. Transformations for the six axis wrist
This section discusses two methods of generating configuration options. The first method systematically generates options within a local hypercube about the robot's current configuration. The second method bases the configuration options on a simulated annealing algorithm and thus incorporates randomness.

Perturbing the joint displacements a small amount, $\Delta \theta$, from their current values, $\underline{\theta}$, generates a set of local configuration options:

$$
\underline{\hat{\theta}}: \underline{\hat{\theta}}=\underline{\theta}+\underline{\varepsilon} \Delta \theta
$$

where $\underline{\varepsilon}$ is an arbitrary sweep vector with all elements equal to $\pm 1$ or 0 . The vector of current displacement values, $\underline{\theta}$, is the base point for the perturbations. At the base point, $\underline{\varepsilon}=\underline{0}$. All other $\underline{\varepsilon}$ with elements equal to combinations of $\pm 1$ and 0 generate points on the faces, edges, and vertices of an $n$-dimensional hypercube with $n$ equal to the number of joints involved in the exploration.


Figure 2. shows the hypercube for a robot with three degrees of redundancy. There are $2 n$ points on the faces
of the cube, $2^{n}$ points at the vertices, and $3^{n}$ points in all. Respectively, we call these the simple, factorial, and exhaustive exploration patterns. For more than four or five degrees of redundancy, the computational expense associated with current computing hardware prohibits the exhaustive pattern in real-time applications.

Annealing describes a process of heating a material to an elevated temperature and then cooling it very slowly. The slow cooling allows the material to reach a low energy state in which it is relatively ductile. With no intelligence or systematic strategy, some materials minimize energy state during the slow cooling. Simulated annealing is an approximation of this natural process carried out on a computer and is based on the Boltzmann probability distribution.

$$
\operatorname{Prob}(E) \approx \exp \left(-\frac{E}{k T}\right)
$$

In this equation, $E$ is the energy of the system, $k$ is Boltzmann's constant, and $T$ is the temperature. Essentially, the Boltzmann probability distribution states that a system's energy is probabilistically distributed depending upon the temperature. As the temperature increases, the probability of the system assuming a higher energy state increases. As the temperature is lowered, the odds of the system leaving a lower energy state decrease. Each configuration option corresponds to an energy state. Because simulated annealing algorithms sometimes leave lower energy states for higher ones, they can escape from local minima. Simulated annealing algorithms typically include a method of generating random changes in the system's configuration. The random changes represent trial configurations evaluated using the Boltzmann probability distribution. If the distribution indicates, the system assumes the trial configuration; otherwise it is discarded.

## Performance Criteria

We have formulated and implemented in software over 30 performance criteria (Van Doren and Tesar, 1992). These criteria emphasize task-based performance indicators derived from the physical description of the manipulator. These formulations emphasize efficiency and portability. With currently available computational hardware, decisions based on several of these criteria are possible in real-time. Given the rapid pace of advancements in computational speed, we feel that it will soon be possible to employ the entire suite of performance criteria in a realtime decision making process. Table 1 . lists the general categories of these performance criteria. Our continuing
work focuses on issues of normalization and multiple criteria fusion.

| Table 1. General categories of performance criteria. |  |
| :--- | :--- |
| Category | Characteristics |
| constraint criteria | physical limitations |
| geometric | task independent |
| inertial | from dynamic models |
| compliance | design and operational issues |
| kinetic energy | content and distribution |

Elementary physical limitations form the basis for the constraint criteria. These limitations restrict joint travels, joint speeds, joint accelerations, and joint torques. The joint travel availability is a representative criterion that seeks to keep the joint displacements as near as possible to the midpoints of their travel.

The Jacobian matrix forms the basis for the geometric performance criteria. These criteria are task independent and based only on the geometry of the robot, thus these criteria are formulated once for each robot with no need for reformulation if the task changes (Cleary and Tesar, 1990).

The inertial performance criteria have their basis in dynamic models of forces and torques within the robot and are essential to the intelligent design and application of robots. The rate of change of inertial criteria measure how fast the robot can respond to torque and force demands. They are especially important because larger actuators or higher gear ratios can supply more torque, but both will slow the overall response of the robot to external disturbances.

The compliance criteria describe the robot's ability to perform precision operations under load. They also correspond to the vibratory modes of the robot. Of the compliance criteria, the potential energy partition values, are particularly important. The potential energy partition values measure the distribution of compliance energy and how it changes as the robot moves. An unusually high compliance energy content in any part of the robot indicates a problem with the robot's design. Rapid changes in compliance energy indicate large local forces, which correspond to large actuator demands and decreased precision.

The kinetic energy performance criteria address high-level issues represented in relatively simply formulations. Large changes in kinetic energy correspond to very large
demands on actuator power. Very rapid changes in the kinetic energy represent shocks to the robot.

## Dual-Arm Robot Example

This example illustrates the application of the motion coordination method described above as applied to the Dual Arm Work Module (DAWM) designed and recently demonstrated at Oak Ridge National Laboratory (Figure 4.). The DAWM is a dual-arm manipulator system designed to perform an extremely wide variety of tasks, thus amortizing development costs. These tasks include disassembly of process equipment, cutting pipes, size reduction of equipment, transport of materials, and decontamination of floors, walls, and remaining equipment. The DAWM has 17 DOF arranged in 2 serial chains each having 8 independent DOF and sharing 1 common center rotational joint. Schilling Titan II manipulators form the last six DOF for each arm.


Figure 3. The Dual-Arm Work Module
The example begins with a reverse position analysis of the Schilling Titan II manipulators. Figure 4. shows a schematic of the Schilling arm. The offset at the wrist prevents the last three joint axes from intersecting at a point (a spherical wrist), thus precluding a reverse position solution that simply decouples the positional and orientational constraints. The following analysis provides a solution involving polynomials of degree 2 of less. The analysis follows three basic steps. The first step solves for $\Phi$ and $\theta_{1}$ using the positional $\left(P_{x}, P_{y}, P_{z}\right)$ and orientational $(\alpha, \beta, \gamma)$ constraints. The next step uses $\Phi$ and $\theta_{1}$ to remove the effects of the wrist offset. After
this, step three solves for $\theta_{2}$ through $\theta_{6}$ as if the robot had a spherical wrist.


Figure 4. Schematic of the Schilling Titan II
The analysis begins by finding $\theta_{1}$. Since $\theta_{2}, \theta_{3}$, and $\theta_{4}$ are in parallel:

$$
\theta_{1}=\mathrm{a} \tan 2\left(P_{y}, P_{z}\right)
$$

Using $Y X Z$ Euler angles to match the rotations at the wrist:

$$
\begin{aligned}
& { }_{6}^{0} R=R_{y}(\alpha) R_{x}(\beta) R_{z}(\gamma) \\
& { }_{\Phi}^{0} R=R_{x}\left(-\theta_{1}\right) \\
& { }_{6}^{0} R={ }_{\Phi}^{0} R_{6}^{\Phi} R \\
& { }_{6}^{\Phi} R={ }_{\Phi}^{0} R^{-1}{ }_{6}^{0} R .
\end{aligned}
$$

Extracting $Y X Z$ Euler angles from ${ }_{6}^{\Phi} R$ gives:

$$
\Phi=\alpha
$$

By finding $\Phi$ and $\theta_{1}$, this completes the first step in the analysis.

The next step in the analysis uses $\Phi$ and $\theta_{1}$, to eliminate the effects of the wrist offset, $L_{4}$, and transforms $P_{x}, P_{y}, P_{z}$ into $P_{x}^{\prime}, P_{y}^{\prime}, P_{z}^{\prime}$. The transformation proceeds as follows:

$$
\begin{aligned}
& P_{x}^{\prime}=P_{x}-L_{4} \sin \Phi \\
& P_{y}^{\prime}=P_{y}-L_{4} \cos \Phi \sin \theta_{1} \\
& P_{z}^{\prime}=P_{z}-L_{4} \cos \Phi \cos \theta_{1} .
\end{aligned}
$$

This transformation essentially subtracts the effects of the offset.

Given $P_{x}^{\prime}, P_{y}^{\prime}, P_{z}^{\prime}$, the final step in the procedure solves for the joint displacements as is the robot had a spherical wrist. The forward position solution for the transformed geometry generates the following geometric equations:

$$
\begin{aligned}
& P_{x}^{\prime}=L_{2} \sin \theta_{2}+L_{3} \sin \left(\theta_{2}+\theta_{3}\right) \\
& P_{y}^{\prime}=L_{1} \sin \theta_{1}+L_{2} \sin \theta_{1} \cos \theta_{2}+L_{3} \sin \theta_{1} \cos \left(\theta_{2}+\theta_{3}\right) \\
& P_{z}^{\prime}=L_{1} \cos \theta_{1}+L_{2} \cos \theta_{1} \cos \theta_{2}+L_{3} \cos \theta_{1} \cos \left(\theta_{2}+\theta_{3}\right)
\end{aligned}
$$

Substituting for the known $\theta_{1}$ and rearranging produces two equations in two unknowns of the form:

$$
\begin{aligned}
& c=a \cos \left(\theta_{2}+\theta_{3}\right)+b \cos \theta_{2} \\
& d=a \sin \left(\theta_{2}+\theta_{3}\right)+b \sin \theta_{2}
\end{aligned}
$$

Paul (1981) shows the solution for $\theta_{3}$ as:

$$
\theta_{3}=\operatorname{atan} 2\binom{ \pm \sqrt{1-\left(\frac{c^{2}+d^{2}-a^{2}-b^{2}}{2 a b}\right)^{2}}}{\frac{c^{2}+d^{2}-a^{2}-b^{2}}{2 a b}}
$$

Substituting $\theta_{3}$ into the forward position equations yields two equations in one unknown of the form:

$$
\begin{aligned}
& g=e \cos \theta_{2}-f \sin \theta_{2} \\
& h=e \sin \theta_{2}+f \cos \theta_{2}
\end{aligned}
$$

Wolovich (1987) shows these equations have the solution:

$$
\theta_{2}=\mathrm{a} \tan 2(e h-f g, e g+f h)
$$

Again because $\theta_{2}, \theta_{3}$, and $\theta_{4}$ are in parallel:

$$
\theta_{4}=\Phi-\theta_{2}-\theta_{3} .
$$

The remaining unknowns are $\theta_{5}$ and $\theta_{6}$. Because the axes of rotation for these angles intersect at a point, the following Euler angle extraction process at the point of intersection will find $\theta_{5}$ and 6 .

$$
\begin{aligned}
& { }_{6}^{0} R=R_{y}(\alpha) R_{x}(\beta) R_{z}(\gamma) \\
& { }_{4}^{0} R=R_{x}\left(-\theta_{1}\right) R_{y}(\Phi) \\
& { }_{6}^{4} R={ }_{4}^{0} R^{-1}{ }_{6}^{0} R .
\end{aligned}
$$

Extracting $Y X Z$ Euler angles from ${ }_{6}^{4} R$ completes the solution for $\theta_{1}$ through $\theta_{6}$.


Figure 5. shows a computer generated snapshot of the dual-arm robot operating in an environment strewn with a number of pipe-like obstacles. Using the simulated annealing method described above, the motion coordination method cycles at over 100 Hertz on a modest personal computer.


Figure 6. Simulated trace of straight line path

Figure 6. shows a computer generated trace of one arm of the robot following a straight-line path. With only one arm ( 9 DOF ) and the simple environment, the motion coordination algorithm cycles at rates in the hundreds of Hertz on a Pentium 100Mhz personal computer

## Conclusions

A robot is a complex multiple-input and multiple-output system. This paper presented the position that multiple performance criteria must assess the performance of a robot. Though the paper offered no proof of this, consider the errors commonly encountered when programming an industrial robot, including: joint travel limits, joint speed limits (often reflecting singularities), motor current overloads, and workspace limits. An experienced operator will consider these and other limitations (such as obstacles) when programming the robot. A reasonable motion coordination method should at least match this level of expertise. Ultimately, higher-level criteria addressing issues of geometry, force, compliance, and energy will refine the motion and further enhance performance.

As a basis for motion coordination, this paper presented and discussed a number of categories for performance criteria. These criteria emphasize task-based performance indicators derived from the physical description of the manipulator. The origins of these criteria are from foundation activity in high speed mechanisms for production machinery (Benedict and Tesar, 1978). There, the issues of precision and modeling of complex nonlinear structures forced the development of a geometric understanding for mechanical structures and how to represent them with efficient analytical tools. Thomas and Tesar (1982) showed that the concept of kinematic influence coefficients (used in systems with 1 DOF) were effective in spatial manipulator structures with N DOF. The five basic categories for these measures are: constraint, geometry, inertial, compliance, and kinetic energy.

This work outlined a method of motion coordination combining closed-form reverse position analysis, local exploration, and multi-criteria decision making. The closed-form reverse position analysis satisfies the placement constraints on the robot's EEF (inverse kinematics). Using closed-form reverse position analysis leverages over two decades of work by a number of dedicated scholars. The local explorations generate a set of motion options for evaluation by the decision making process. Finally, the decision making process evaluates
the options based on a series of performance criteria and identifies one as the next set-point command for the robot's servo controllers.

This method of redundancy resolution has been tested (either in simulation or experimentation) on robots with $7,8,9,10,17$, and 21 DOF . This paper presented simulation results for a dual-arm robot with 17 DOF. The results show successful motion coordination incorporating multiple criteria at a rate of over 100 cycles per second on a Pentium 100 personal computer.

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