

Chapter 1: Introduction

The goal when this work began was the development of an **inverse kinematics*** method for modular robots. Modular robots may assume an almost limitless number of different geometric configurations. Thus, generality with respect to the robot's geometry was requisite. There was also a growing belief that advanced robots would incorporate a wide variety of **performance criteria** during their operation. This led to the specification that the inverse kinematics method should consider performance criteria and include them in a general fashion as well. The insistence on generality led this work into becoming much more than an inverse kinematics method for modular robots. This dissertation details an inverse kinematics method applying to all **serial robots**.

Figure 1.1 shows a serial robot. A serial robot is a single chain of joints connected by links. When the joint displacements change, the links move relative to one another. A general serial robot has no a priori constraints on the size or shape of the links. The inverse kinematics problem for a serial robot is: given the position and orientation of the last link relative to the first link, find the joint displacements. There is almost always more than one set of joint displacements satisfying the position and orientation specifications. The robot's controller must choose a single solution based on one or more performance criteria.

* The glossary defines words appearing in bold.

The robot's last link is its **end-effector** and its first link is its base. The motion of the end-effector relative to the base specifies the robot's task. The robot moves its joints to move the end-effector and accomplish its task. Inverse

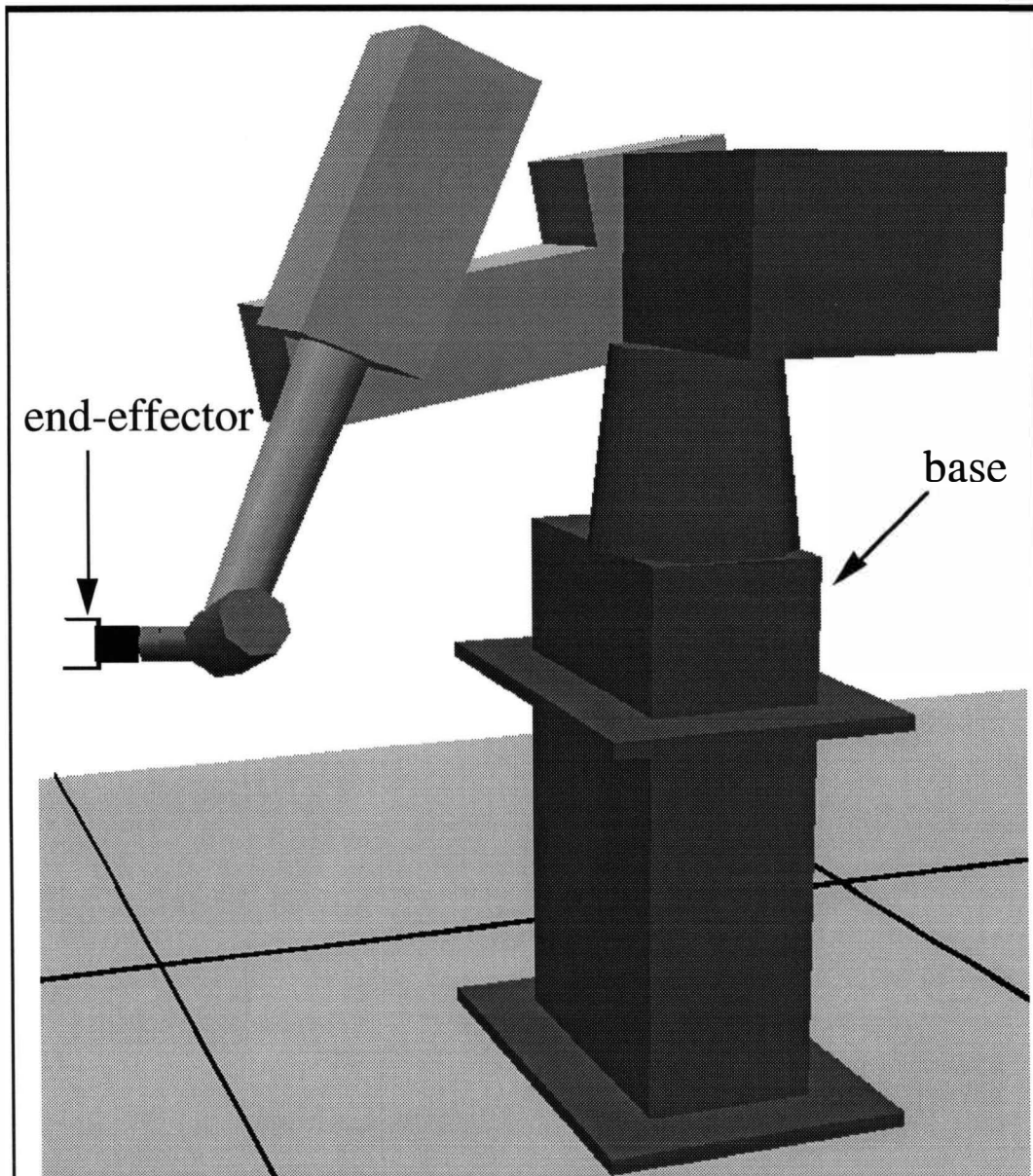


Figure 1.1 A serial robot.

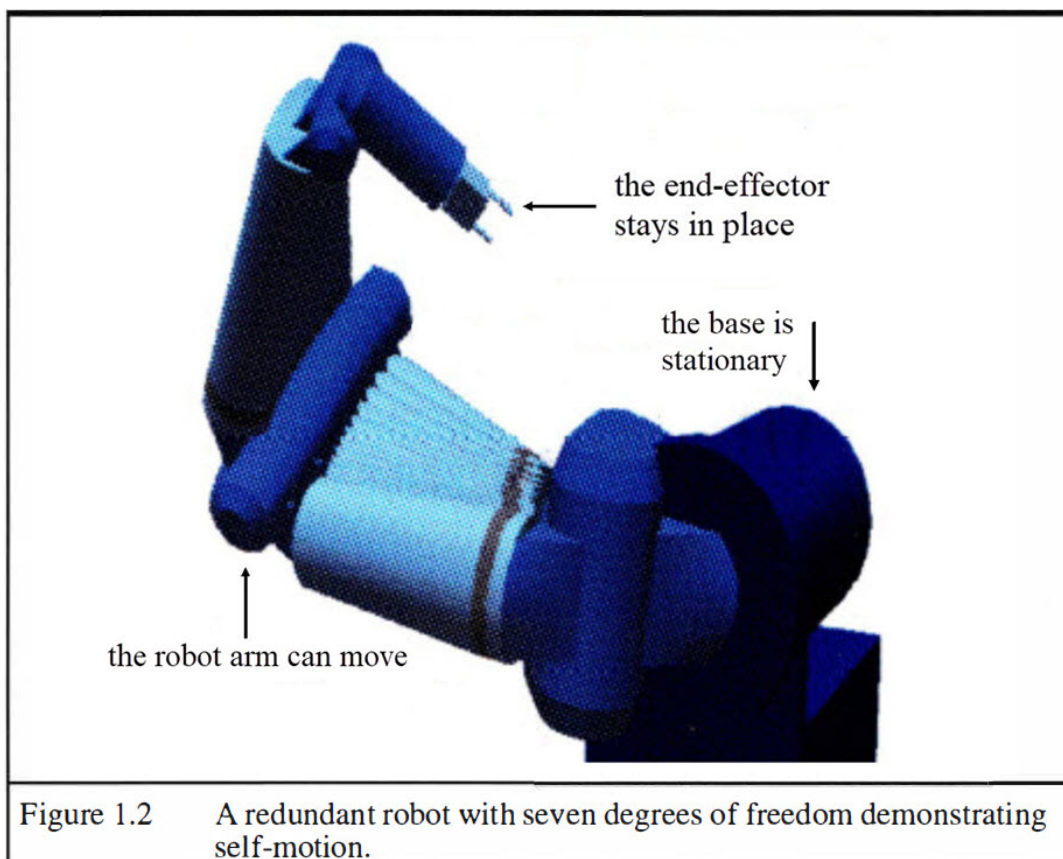
kinematics translates the task specification from end-effector motion to joint motion. A robot cannot do useful work without some form of inverse kinematics.

1.1 INVERSE KINEMATICS

Position and orientation constraints specify the placement of the end-effector relative to the base. The inverse kinematics problem is to find the set of joint displacements satisfying these constraints. There are two different types of robot joints. A revolute joint turns about an axis. An angle specifies the displacement of a revolute joint. A prismatic joint slides along an axis. A length specifies the displacement of a prismatic joint. All other types of robot joints may be described by various combinations of these two joints. Every joint gives the robot a degree of freedom. The different joints and the size and shape of the links determine the robot's geometry. Some inverse kinematics techniques solve for the change in joint displacements with respect to time. These **velocity-level** solutions find the joint speeds and then integrate them once to get the joint displacements. Other inverse kinematics techniques solve for the change in joint speeds with respect to time and then integrate twice to get the joint displacements. These are **acceleration-level** solutions. **Position-level** solutions find the joint displacements directly.

A **fully constrained robot** has as many **degrees of freedom** as there are constraints on the placement of the end-effector. There are a finite number of solutions to the inverse kinematics problem for a fully constrained robot. A robot with more degrees of freedom than constraints is redundant. There can be at most six constraints, three on the position and three on the orientation, on the placement

of the end-effector so any robot with more than six degrees of freedom is redundant. There is typically an infinite number of solutions to the inverse kinematics problem for a redundant robot. Thus, a redundant robot can hold the placement of the end-effector constant while still moving the intermediate links. This is called **self-motion**. Figure 1.2 shows a redundant robot with seven degrees of freedom exhibiting self-motion.



For redundant robots, the inverse kinematics problem is also a type of **optimization** problem. The inverse kinematics algorithm uses performance criteria to choose the best solution. Performance criteria are based on kinematic

and dynamic models of the robot. Often criteria measure the performance of each joint individually. For the whole robot, these criteria are vector quantities. A norm expresses these vector quantities as scalars. The **two-norm** is the square root of the sum of the squares, which is the magnitude of the vector.

Singularities are extremely important in inverse kinematics. A **singularity** occurs when two or more of the robot's joint motions are **linearly dependent**. In this case the normal method of mathematical inverse fails due to a reduction in structure; i. e., a row in the matrix describing the system's input to output relationships disappears. This is true any time two revolute or two prismatic joint axes are collinear. The joints involved in the singularity are no longer independent degrees of freedom. Most inverse kinematics solutions are undefined at singularities. The **direct search** method developed in this dissertation does not suffer from this problem. A fully constrained robot loses its ability to move the end-effector in at least one direction at singularities. Redundant robots lose at least one degree of **redundancy** at singularities, but because they have extra degrees of freedom, they still may be able to move the end-effector in all directions.

Modular robots are made from self-contained joints with one, two, or three degrees of freedom and generic links (Tesar and Butler, 1989a). Each joint or link is a module. The modules may be assembled into a large number of different robot configurations. Tesar and Butler describe a wide variety of benefits associated with modular robots. Because modular robots may assume a large number of different configurations, they demand a general inverse kinematics approach.

Generalized inverse kinematics addresses the inverse kinematics problem in the most general fashion. A generalized formulation of the inverse kinematics problem must make no a priori constraints on the robot's geometry or on the number or type of performance criteria treated by the solution. This dissertation formulates direct search as a solution to the generalized inverse kinematics problem.

1.2 OVERVIEW

The introduction presents a definition of the inverse kinematics problem and discusses its fundamental importance to robotics. The definition and discussion introduce the terminology used throughout this dissertation.

Chapter 2 is a critical literature review. This review describes, and discusses the merits of, important work presented in the literature. By tracing the development of inverse kinematics from its origins in mechanism theory to its present application with redundant robots and optimization, this chapter also provides some historical perspective. This chapter covers both **analytical** and **numerical methods**.

Chapter 3 begins by deriving several analytical solutions to the inverse kinematics problem using optimization methods. Each of these derivations employs **LaGrange multipliers** to express the constrained problem as an unconstrained problem. The derivations use calculus for the actual minimization. The chapter then develops an analogous numerical optimization method for solving the inverse kinematics problem. The method expresses the inverse kinematics problem as an unconstrained problem using an **error function** to

transform the position and orientation constraints on the end-effector. Direct search minimizes the error function. By minimizing the error to zero, the direct search finds a solution to the inverse kinematics problem. This method is general with respect to the robot's geometry and stable at singularities. Development of the direct search method and its successful application to the inverse kinematics problem is the primary contribution of this work.

Chapter 4 extends the direct search method to address the multicriteria inverse kinematics problem. The chapter begins by defining the multicriteria inverse kinematics problem as an optimization problem with an unlimited number of performance criteria, **inequality constraints**, and **equality constraints**. The definition is general and concise. Specifically, it does not constrain the definition of optimality. This chapter discusses two different types of optima and develops direct search methods of finding both. The direct search method can explicitly calculate and consider any number and variety of performance criteria during the solution. The only constraint is that the performance criteria must be single-valued functions of the joint displacements and/or their derivatives with respect to time.

Chapter 5 details the systematic design of an automatic **plant description** system for robots. The plant description is essentially a dynamic model of the robot. The design process loosely follows the method of Pahl and Beitz (1988). This design method leaves a structured record of the goals, constraints, and specifications that influenced the design. In its current form, this plant description system calculates performance criteria values, joint forces or torques, and deflections of the robot's end-effector. The system is automatic because it

produces its outputs given its inputs with no further user intervention. A generalized inverse kinematics implementation must include a plant description system. Generalized inverse kinematics loses many of its associated benefits if the plant description system is not general and automatic. The automatic plant description system is also useful for: designing robots, choosing monolithic robots, configuring modular robots, and enhancing robot control.

Chapter 6 presents experimental and simulation results for several applications of the direct search inverse kinematics method. The robots in these examples have especially difficult or interesting geometries. They cover a spectrum, from an industrial robot with six degrees of freedom to a conceptual robot with twenty-one degrees of freedom. Most of the examples include a graph showing the accuracy of the inverse kinematics solution and timing results for the solution speed.

Chapter 7 maps the future of generalized inverse kinematics. This chapter is a road map for a road that does not yet exist. Mapping roads into the future is a risky activity. This map is based on ten advanced robot applications with the hope that applications will encourage relevance into the future. Each application possesses a degree of difficulty that likely requires generalized inverse kinematics. Applying generalized inverse kinematics, in turn, requires an array of supporting technology. This chapter discusses ten supporting technologies as they relate to generalized inverse kinematics and the application of advanced robots.

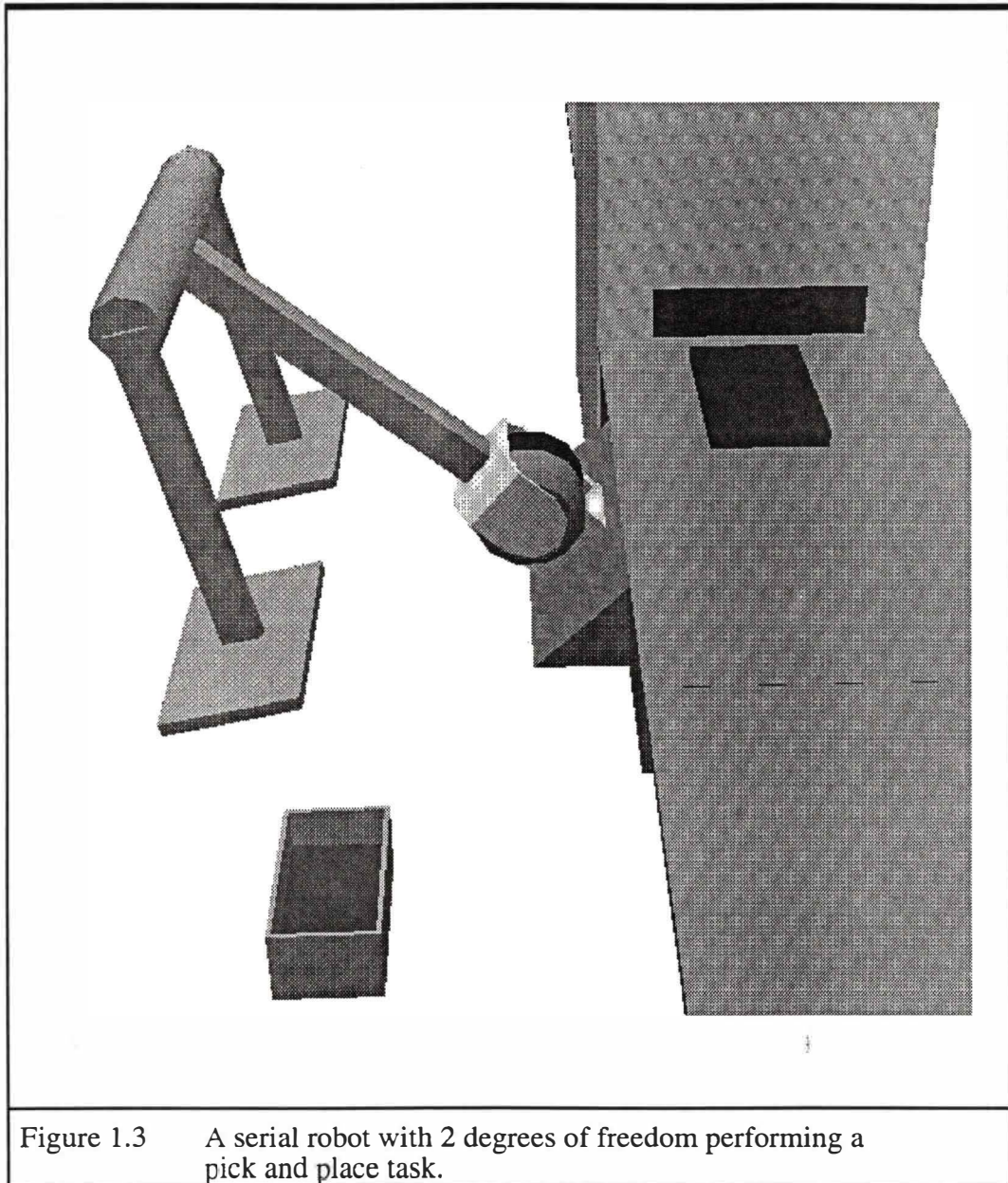


Figure 1.3 A serial robot with 2 degrees of freedom performing a pick and place task.



Figure 1.4 A common industrial robot often used for welding.